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Modeling of Double Star Induction Motors with Dynamic Eccentricity

Zohra yasmine Kecita [1], Mohamed Yazid Kaikaa [1] and Redjem Rebbah[1]

[1] Department of Electrotechnique , University of Frère Mentouri Constantine1, Constantine, Algeria

***Abstract*:** *Like in the three-phase case, double star induction motor (DSIM) may be the subject of various defects like broken rotor bars , bearing defects, loss of phase , eccentricity problems and stator inter-turn short circuit. In this paper a new analytical model of double star induction machine (DSIM) with dynamic eccentricity (DE) is proposed. Using mixture of modified winding function and Fourier series approaches, all inductances of the motor are calculated analytically as closed algebraic equation versus geometrical and electrical parameters. This eliminates the need for large look-up tables and numerical differentiations, which are used frequently in electrical machines and speeds up significantly the computer simulation time.*

Keywords: *Double star induction machine, dynamic eccentricity, modelling, modified winding function, Fourier series, stator current.*

1. Introduction

Multiphase induction motor, especially double star induction motor (DSIM), has found application in high power industry drives because it offers more advantages than the classical three-phase one. The most important advantage of (DSIM) is the power segmentation which reduces each power electronic device in multi phase inverter system. Another advantage is the possibility of fault tolerant drive when a phase is lost due a failure on a power electronic switch [1].

Like in the three-phase case, (DSIM) may be the subject of various defects like broken rotor bars [2], bearing defects[3], loss of phase [4], eccentricity problems [5] and stator inter-turn short circuit [6]. Rotor eccentricity is defined as a condition of the asymmetric air gap that exists between the stator and rotor. There are three types of air-gap eccentricity: static, dynamic and mixed eccentricity. In the case of static eccentricity (SE), the position of the minimal radial air-gap length is fixed in space, while in the case of dynamic eccentricity (DE), the centre of the rotor is not at the centre of the rotation and the position of the minimum air-gap rotates with the rotor.

Different methods have been introduced to modelling DSIM the most efficient ones are based on finite element (FE) analysis [7]. They provide the most accurate results for prediction of the DSIM performance because they are able to obtain the magnetic field distribution using all its magnetization characteristics. .However, these methods are very complicated and analysis is too time consuming process. An interesting method called the modified winding function approach (MWFA) has been developed in recent years taking into account the geometry and winding layout of the machine. This method has the advantage that it offers the possibility to derive analytical expressions of self and mutual inductances of stator and rotor windings.

In this paper we propose a new analytical model of DSIM with dynamic eccentricity (DE). Using mixture of winding function and Fourier series approach, all inductances of the DSIM are computed as closed algebraic equation versus geometrical and electrical parameters. This eliminates the need to interpolate the data for inductance calculation in the simulation process and speeds up significantly the computer simulation time.

2. Air-gap Permance Modelling

In the case of DE illustrated in Fig. 1, the rotor rotational center is different from its geometrical center but identical to the geometrical stator center. The air gap length varies as a function of θ_s and the mechanical rotor angle; it can be expressed as [8]:

$$g(\theta,\theta_s)^{-1} = \frac{1}{g_0}(1+\delta_d \cos(\theta-\theta_s)) \quad (1)$$

Where : g_0 is air gap length in machine with uniform air gap

δ_d : is the degree of dynamic eccentricity, it is defined by the ratio $\delta_d = \frac{O_s O_r}{g_0}$

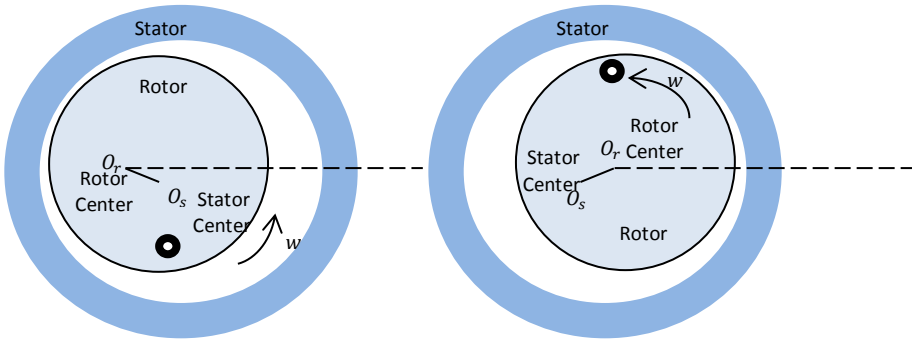

Fig. 1 Illustration of DE. The Right Hand Figure Indicates the Initial Position, the Left Hand Figure Indicates the Position After Half Rotor Turn

3. Modeling of Double Star Induction Machine

The stator of the motor considered in this study has six phases divided into two sets of symmetrical three-phase winding (with phase shift between phases of 120º) separated by an angle $(\alpha = 30^{\circ})$ figure 2. The rotor is a squirrel cage type. Multiple coupled circuit modelling for the DSIMs includes six differential voltage equations for the stator windings, Nb+1 differential voltage equations for the rotor meshes, and two mechanical differential equations. Thus, the model can be represented in the matrix form by:

$$[V_{s1}] = [R_{s1}] \times [I_{s1}] + \frac{d}{dt}[\psi_{s1}] \quad (2)$$

$$[V_{s2}] = [R_{s2}] \times [I_{s2}] + \frac{d}{dt}[\psi_{s2}] \quad (3)$$

$$[V_r] = [R_r] \times [I_r] + \frac{d}{dt}[\psi_r] \quad (4)$$

$$[V_{s1,2}] = [V_{sa1,2} \quad V_{sb1,2} \quad V_{sc1,2}]^T \quad (5)$$

$$[I_{s1,2}] = [I_{sa1,2} \quad I_{sb1,2} \quad I_{sc1,2}]^T \quad (6)$$

$$[V_r] = [V_{r1} \quad V_{r2} \quad \ldots . V_{rnb} \quad V_{re}]^T \quad (7)$$

$$[I_r] = [I_{r1} \quad I_{r2} \quad \ldots . I_{rnb} \quad I_{re}]^T \quad (8)$$

$$[\psi_{s1,2}] = [\psi_{sa1,2} \quad \psi_{sb1,2} \quad \psi_{sc1,2}]^T \quad (9)$$

$$[\psi_r] = [\psi_{r1} \quad \psi_{r2} \quad \ldots . \psi_{rnb} \quad \psi_{re}]^T \quad (10)$$

$$[R_{s1,2}] = \text{diag}\,[r_{sa1,2} \quad r_{sb1,2} \quad r_{sc1,2}] \quad (11)$$

Where: $[V_{s1,2}]$is the stator voltages vector, $[I_{s1,2}]$is the stator currents vector, $[I_r]$ is the rotor loops currents vector, $[R_{s1,2}]$ is the stator windings resistance matrix and $[\psi_{s1,2}]$, $[\psi_r]$ are the total flux linkage by stator and rotor windings, respectively.

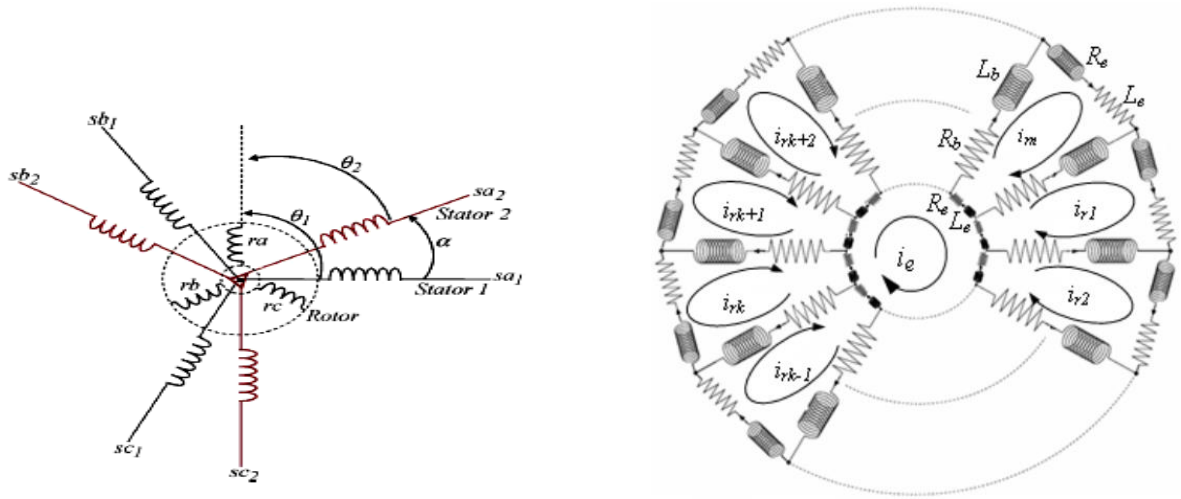

Fig 2. Right hand figure: Rotor Loops. The Left-Hand Figure Winding of the DSIM

The matrix$[R_r]$ is the rotor resistances matrix where, Re is the end ring segment resistance, and the Rb the rotor bar resistance.

$$[R_r] = \begin{bmatrix} 2(R_b+R_e) & -R_b & 0 & 0 \dots 0 & -R_b & -R_e \\ -R_b & 2(R_b+R_e) & -R_b & 0 \dots 0 & 0 & -R_e \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -R_b & 0 & 0 & 0 \dots -R_b & 2(R_b+R_e) & -R_e \\ -R_b & -R_e & -R_e & -R_e \dots -R_e & -R_e & n_b R_e \end{bmatrix} \quad (12)$$

Assuming linear magnetic system, the flux-linkages vectors are calculated using the current vectors and the inductance matrices as follows:

$$\begin{bmatrix} [\psi_{s1}] \\ [\psi_{s2}] \\ [\psi_r] \end{bmatrix} = \begin{bmatrix} [L_{ss1}] & [L_{s12}] & [L_{sr1}] \\ [L_{s21}] & [L_{ss2}] & [L_{sr2}] \\ [L_{rs1}] & [L_{rs1}] & [L_{rr}] \end{bmatrix} \times \begin{bmatrix} [I_{s1}] \\ [I_{s2}] \\ [I_r] \end{bmatrix} \quad (13)$$

Where [Lss] and [Lrr] include self/mutual inductances of the stator windings and the rotor meshes respectively:

$$[L_{ss1,2}] = \begin{bmatrix} L_{ma1,2} + L_{fa1,2} & L_{mb1,2}.\cos\left(\frac{2\pi}{3}\right) & L_{mc1,2}.\cos\left(\frac{4\pi}{3}\right) \\ L_{mc1,2}.\cos\left(\frac{4\pi}{3}\right) & L_{mb1,2} + L_{fb1,2} & L_{ma1,2}.\cos\left(\frac{2\pi}{3}\right) \\ L_{ma1,2}.\cos\left(\frac{2\pi}{3}\right) & L_{mc1,2}.\cos\left(\frac{4\pi}{3}\right) & L_{mc1,2} + L_{fc1,2} \end{bmatrix} \quad (14)$$

$$[L_{s12}] = \begin{bmatrix} L_{ms}.\cos(\alpha) & L_{ms}.\cos\left(\alpha+\frac{2\pi}{3}\right) & L_{ms}.\cos\left(\alpha+\frac{4\pi}{3}\right) \\ L_{ms}.\cos\left(\alpha+\frac{4\pi}{3}\right) & L_{ms}.\cos(\alpha) & L_{ms}.\cos\left(\alpha+\frac{2\pi}{3}\right) \\ L_{ms}.\cos\left(\alpha+\frac{2\pi}{3}\right) & L_{ms}.\cos\left(\alpha+\frac{4\pi}{3}\right) & L_{ms}.\cos(\alpha) \end{bmatrix} \quad (15)$$

With $$L_{ma1} = L_{mb1} = L_{mc1}\ ;\ L_{ma2} = L_{mb2} = L_{mc2}\ ;\ L_{ms} = L_{ma1} = L_{ma2} \quad (16)$$

$$[L_{rr}] = \begin{bmatrix} L_{rp} + 2L_b + 2\frac{L_e}{n_b} \dots & M_{rr} & M_{rr} & M_{rr} \dots \\ \vdots & \vdots & \vdots & \vdots \ \dots \\ M_{rr} & M_{rr} - L_b & L_{rp} + 2L_b + 2\frac{L_e}{n_b} & M_{rr} - L_b \dots \\ \vdots & \vdots & \vdots & \vdots \\ M_{rr} - L_b & M_{rr} & M_{rr} & M_{rr} \dots \end{bmatrix} \quad (17)$$

Also, [Lsr] and [Lrs] include mutual inductances between the stator phases and the rotor meshes as follows:

$$[L_{sr1;2}] = \begin{bmatrix} L_{a1;2\,r1} & L_{a1;2r2} & \dots & L_{a1;2rnb} & L_{a1;2e} \\ L_{b1;2r1} & L_{b1;2r2} & \dots & L_{b1;2rnb} & L_{b1;2e} \\ L_{c1;2r1} & L_{c1;2r2} & \dots & L_{c1;2rnb} & L_{c1;2e} \end{bmatrix} \quad (18)$$

$$[L_{s21}] = [L_{s12}]^T;\ [L_{sr1;2}] = [L_{rs1;2}]^T$$

The equations describing the mechanical part of the system:

$$J\frac{d\omega_r}{dt} + T_e = T_c \quad (19)$$

$$\frac{d\theta}{dt} = \omega_r \quad (20)$$

Where: T_e is the electromagnetic torque produced by the motor. J is rotor inertia, ω_r is rotor angular speed, T_c is the load torque. T_e Can be computed as:

$$T_e = \left[\frac{\delta W_{co}}{\delta\theta}\right]_{I_s, I_r const} \quad (21)$$

$$W_{co} = \frac{1}{2}\{[I_{s1}]^T.[\psi_{s1}] + [I_{s2}]^T.[\psi_{s2}] + [I_r]^T.[\psi_r]\} \quad (22)$$

Above system of equations could be easily solved using some of the numerical techniques for known parameters of the model. So, the next step is to calculate the inductance matrix.

4. Modified Winding Function and Inductances Calculation

According to [9] inductance between any two windings "s" and "r" can be computed by the following equation:

$$L_{sr}(\theta) = \mu_0 rl \int_0^{2\pi} n_{rk}(\theta_r, \theta) M_{sq}(\theta_s, \theta) g(\theta_s, \theta)^{-1} d\theta_s \quad (23)$$

where: θ_s is the angular position of the rotor with respect to some rotor reference θ is a particular angular position along the stator inner surface, L is the length of stack and r is the average radius of air gap.
$n_{rk}(\theta_r, \theta)$ is the winding distribution of Kth rotor loop $M_{sq}(\theta_s, \theta)$ is the modified winding function (MWF) of phase "q", $g(\theta_s, \theta)^{-1}$ is the inverse gap.

4.1. Derivation of Stator Inductances

According to [10] the turn function of the stator phase "q" of three phase induction machine can be defined by:

$$n_{sq}(\theta_s) = C + \frac{2N_t}{p\pi}\sum_{h=1}^{\infty}\frac{k_{wh}}{h}.\times\cos\left(hp\left(\theta_s - \theta_0 - q\frac{2\pi}{3p}\right)\right) \quad (24)$$

For an (DSIM) and by including the angle $\alpha = 30^\circ$ equation (24) is modified as follows:

$$n_{sqi}(\theta_s) = C + \frac{2N_t}{p\pi}\sum_{h=1}^{\infty}\frac{k_{wh}}{h}.\times\cos\left(hp\left(\theta_s - \theta_0 - q\frac{2\pi}{3p} - i\alpha\right)\right) \quad (25)$$

where: K_{wh} winding factor, N_t number of stator turns in series, q = 0,1,2.

p is the number of pole pairs, h is harmonic order, C and θ_0 are constante

$$i = \begin{cases} 0 \text{ for the first stator winding} \\ 1 \text{ for the second stator winding} \end{cases}$$

The modified winding function (MWF) of phase "q" can be expressed by

$$M_{sq}(\theta_s) = n_{sq}(\theta_s) - \frac{1}{2\pi\langle g(\theta_s)\rangle}\int_0^{2\pi} n_{sq}(\theta_s).g(\theta,\theta_s)^{-1}\,d\theta_s \quad (26)$$

For a given motor dimensions, the self magnetizing inductance of any stator phase can be determined by the following equation:

$$L_{smq} = \mu_0 rl \int_0^{2\pi} n_{sq}(\theta_s) M_{sq}(\theta_s) g(\theta, \theta_s)^{-1} \, d\theta_s \quad (27)$$

By replacing $n_{sq}(\theta_s)$, $M_{sq}(\theta_s)$ and $g(\theta_s)^{-1}$ by their expressions, equation (27) became:

$$L_{smqi} = \frac{\mu_0 rl}{g_0 \pi} \left(\frac{2N_t}{p}\right)^2 \sum_{h=1}^{\infty} \left(\frac{K_{wh}}{h}\right)^2 \left(1 + \delta_d C_0 \cos\left(\theta - \theta_0 - (q-1)\frac{2\pi}{3p}\right)\right) \quad (28)$$

The mutual inductance between any two stator phases is:

$$L_{sq\,s(q+1)} = \mu_0 rl \int_0^{2\pi} n_{sq}(\theta_s) M_{sq+1}(\theta_s) g(\theta_s)^{-1} \, d\theta_s \quad (29)$$

Then, $M_s = L_{sq\,s(q+1)} = \frac{\mu_0 rl}{g_0 \pi} \left(\frac{2N_t}{p}\right)^2 \sum_{h=1}^{\infty} \left(\frac{K_{wh}}{h}\right)^2 \left(1 + \delta_d C_0 \cos\left(\theta - \theta_0 - (q-1)\frac{2\pi}{3p}\right)\right) cos\frac{2\pi}{3}$ (30)

Equations (30) and (32) show clearly that the self and mutual inductances of stator are dependent of the rotor position.

4.2. Derivation of rotor inductances

The turn function of the kth rotor loop is also defined following the same way as in [10] by:

$$n_{rk}(\theta_r) = \frac{1}{n_b} + \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} \sin\left(m\frac{\pi}{n_b}\right) \times \cos\left(m\left(\theta_r - k\frac{2\pi}{n_b}\right)\right) \quad (31)$$

where: $a = \frac{2\pi}{n_b}$ is the mechanical angle of a rotor loop n_b is the number of rotor bars, and k positive integer

By means of equation (26) and by replacing $n_{sq}(\theta_s)$ by $n_{rk}(\theta_r)$, the MWF of the rotor can be expressed as:

$$M_{rk}(\theta, \theta_r) = \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} \sin\left(m\frac{a}{2}\right) \cos\left(m\left(\theta_r - \left(k - \frac{1}{2}\right)a\right)\right) - \frac{\delta_d}{\pi} \sin\left(\frac{a}{2}\right) \cos\left(\theta + \left(k - \frac{1}{2}\right)a\right) \quad (32)$$

The inductance of a rotor loop k is defined by:

$$L_{mrk} = \mu_0 rl \int_0^{2\pi} n_{rk}(\theta_r) M_{rk}(\theta_r) g(\theta_s)^{-1} \, d\theta_r \quad (33)$$

Substituting (1), (31) and (32) into (33), this yields to:

$$L_{mrk}(\theta_r) = \frac{\mu_0 rl}{g_0} \frac{2\pi}{n_b} \left(1 - \frac{1}{n_b}\right) + \frac{\mu_0 rl}{g_0} \frac{\delta_d a}{\pi} \sin\frac{a}{2} \cos\left(\left(k - \frac{1}{2}\right)a\right) - \delta_d^2 \frac{\mu_0 rl}{g_0} \sin\left(\frac{a}{2}\right)^2 \left(1 + \cos 2\left(k - \frac{1}{2}\right)a\right) \quad (34)$$

The mutual inductance between loop "j" and any loop "k" can be obtained by:

$$L_{rjrk} = \mu_0 rl \int_0^{2\pi} n_{rj}(\theta_r) M_{rk}(\theta_r) g(\theta_s, \theta)^{-1} \, d\theta_r \quad (35)$$

By replacing $n_{rj}(\theta_r)$, $M_{rk}(\theta_r)$ and $g(\theta, \theta_s)^{-1}$ by their expressions, equation (37) became:

$$L_{rjk}(\theta_r) = -\frac{\mu_0 rl}{g_0} \frac{2\pi}{{n_b}^2} + \delta_d \frac{2\mu_0 rl}{g_0} \frac{a}{2\pi} \sin\left(\frac{a}{2}\right) \cos\left(k - \frac{1}{2}\right)a - \delta_d^2 \frac{\mu_0 rl}{\pi g_0} \sin\left(\frac{a}{2}\right)^2 \left(1 + \cos 2\left(k - \frac{1}{2}\right)a\right) \quad (36)$$

(19)

4.3. Derivation of Mutual Inductances Stator /Rotor

Substituting (1), (26) and (32) in (23) leads to the general expression of the mutual inductance L_{sr} between a stator winding and a rotor loops. After development we get:

$$L_{sr}(\theta) = \frac{\mu_0 rl}{g_0} \frac{4N_t}{\pi} \left\{ \sum_{h=1}^{\infty} \frac{K_{wh}}{h^2} . \sin\left(hp\frac{a}{2}\right) \times cos\left(hp\theta + hpKa - hp\left(\theta_0 + (q-1)\frac{2\pi}{3p}\right) - i\alpha\right)\right.$$

$$+\frac{p\delta_d}{2}\sum_{h=1}^{\infty}\frac{K_{wh}}{h(hp+1)}.\sin\left((hp+1)\frac{a}{2}\right)\times\ cos\left(hp\theta+(hp+1)Ka-hp\left(\theta_0+(q-1)\frac{2\pi}{3p}\right)-\mathrm{i}\alpha\right)$$

$$+\frac{p\delta_d}{2}\sum_{h=1}^{\infty}\frac{K_{wh}}{h(hp-1)}.\sin\left((hp-1)\frac{a}{2}\right)\ \times cos\left(hp\theta+(hp-1)Ka-hp\left(\theta_0+(q-1)\frac{2\pi}{3p}\right)-\mathrm{i}\alpha\right)\Big\} \quad (37)$$

5. Simulations and Discussions

The MWF model presented in the section 3 has been implemented in the Matlab environment. The double star squirrel cage induction motor used in this paper is 1.1-kW, 220/380, 50 Hz, 4 poles, 36 stator slots and 22 rotor bars. The stator current and its zoom for machine working without and with stator slots effects are shown respectively in figures. 3 and 4.

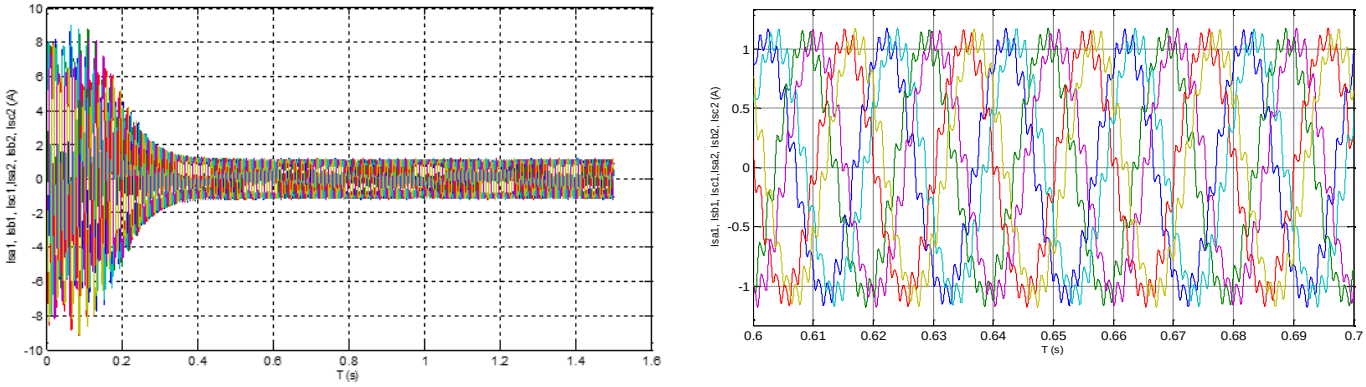

Fig. 3. Stator Line Current and Its Zoom without DE

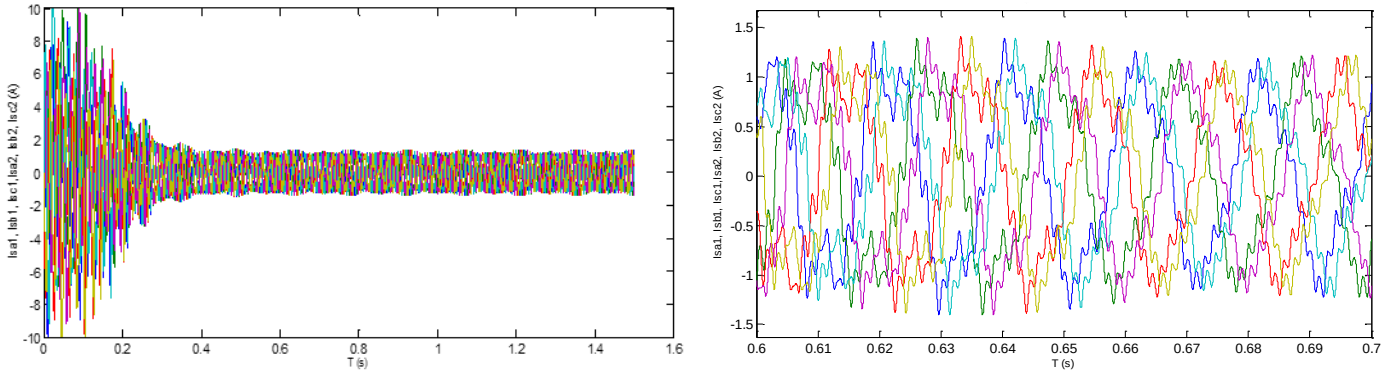

Fig. 4. Stator Line Current and its Zoom with DE

To get more information about the influence of DE, one performs the normalized Fast Fourier Transform (FFT) of the stator current. The results are shown in figures 5 and 6. It can be easily seen the presence of some rotor slot harmonics (RSH) in both cases (with and without DE). However, some of RSH (f_{de1} , f_{de2} and f_{de3}) can be observed clearly only when the DE is taken into account. (see Table 1).

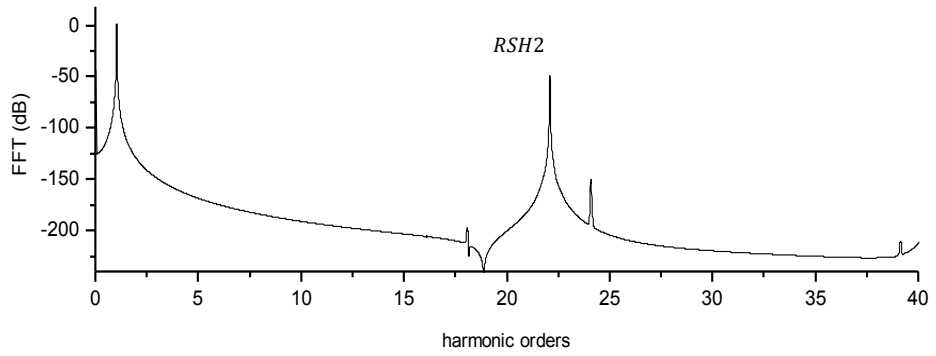

Fig 6. Simulated, Normalized FFT Spectrum of Stator Line Current of an DSIM without Stator Slot Effects

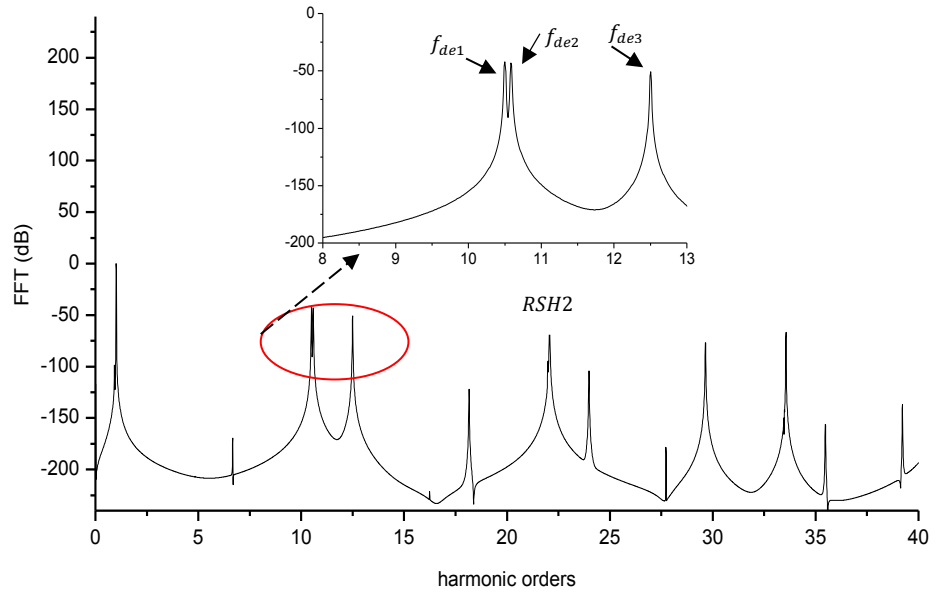

Fig 7. Simulated, Normalized FFT Spectrum of Stator Line Current of an DSIM with DE

TABLE 1: Stator Current Frequency Components at [0 1000 Hz] for s = 0.042

Frequency	Predicted Theoretically [Hz] [Hz]	By Simulation [Hz]
f_{de1}	$\left(\frac{n_b+2}{p}(1-s)-1\right)f_s = 596.4$	596.4
f_{de2}	$\left(\frac{n_b-2}{p}(1-s)+1\right)f_s = 496.4$	496.4
f_{de3}	$f_{de3} = \left(\frac{n_b+2}{p}(1-s)+1\right)f_s = 505.3$	505.4

6. Conclusion

In this paper, an accurate model of double-star induction machine has been presented. This model is based on multiple coupled circuits and takes into account the DE of the rotor. All inductances of the machine have been derived by means of the modified winding function approach (MWFA) and have been integrated with the decomposition into their Fourier series. It has been shown that when DE takes place in DSIM, fault characteristic components appear in the stator current.

7. References

[1] D. Hadiouche, H. Razik, A. Rezzoug, “On the Modeling and Design of Dual-Stator Windings to Minimize Circulating HarmonicCurrents for VSI Fed AC Machines*”, in IEEE Trans.Ind.Applicat., vol. 40, No. 2, pp. 506–516, March/April 2004.*

https://doi.org/10.1109/TIA.2004.824511

[2] L. Alberti and N. Bianchi, “Experimental tests of dual three-phase induction motor under faulty operating condition,” *IEEE Trans. Ind. Electron*, vol. 59, no. 5, pp. 2041–2048, May 2012.

https://doi.org/10.1109/TIE.2011.2171175

[3] S.H. Kia, H. Henao and G-A. Capolino, “A High-Resolution Frequency Estimation Method for Three-Phase Induction Machine Fault Detection”, *in IEEE Trans. on Ind. Electron., vol. 54, no. 4, pp. 2295–2304, Aug. 2007*

https://doi.org/10.1109/TIE.2007.899826

[4] B. Akin, U. Orguner, H. A. Toliyat and M. Rayner, “Low order PWM inverter harmonics contributions to the inverter-fed induction machine fault diagnosis”, in *IEEE Trans. on Ind. Electron.*vol. 55, no. 2, pp. 610-619, Feb. 2008.

https://doi.org/10.1109/TIE.2007.911954

[5] R. N. Andriamalala, H. Razik, L. Baghli, and F.-M. Sargos, “Eccentricity fault diagnosis of a dual-stator winding induction machine drive considering the slotting effects,” *IEEE Trans. Ind. Electron*, vol. 55, no. 12, pp. 4238–4251, Dec. 2008.

https://doi.org/10.1109/TIE.2008.2004664

[6] L. Alberti and N. Bianchi, “Experimental tests of dual three-phase induction motor under faulty operating condition,” *IEEE Trans. Ind. Electron*, vol. 59, no. 5, pp. 2041–2048, May 2012

https://doi.org/10.1109/TIE.2011.2171175

[7] L. Tutelea,S. Deaconu, N. Budisan, I. Boldea “Double stator winding induction generator for wind and hydro applications: 2D-FEM analysis and optimal design” *Power Electronics and Applications (EPE), 2013 15th European Conference on, 1-10.*

[8] D.G. Dorrell, W.T. Thomson and S.Roach, "Analyse of air-gap flux, current, and vibration signals as function of combination static and dynamic air-gap eccentricity in 3-phase induction motors," *IEEE Trans. Industry Applications, vol. 33, no. 1, Jan /Feb 1997, pp. 24-34*

[9] N.A. Al-Nuaim and H.A. Toliyat, "A novel method for modeling dynamic air-gap eccentricity in synchronous machines based on modified winding function theory," *IEEE Trans. Energy Conversion*, vol. 13, no. 2, June 1998, pp. 156-162

https://doi.org/10.1109/60.678979

[10] A. Khezzar, M. Hadjami, N. Bessous, M.E.K. Oumaamar, H Razik, “Accurate Modelling of Cage Induction Machine with Analytical Evaluation of Inductances” 34th Annual Conference of the *IEEE Industrial Electronics Society, IECON’08*, Orlando, Floride, USA, 10-13 Nov. 2008, pp. 1112 - 1117.

https://doi.org/10.1109/IECON.2008.4758110